SPACE CHARGE RESONANCES IN HIGH-INTENSITY BEAMS

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Abstract

In this paper we give an overview on space charge driven response of particle motion in high intensity beams primarily based on self-consistent particle simulation. We focus on transverse space charge effects, including the possibility of longitudinal-transverse exchange. The resonant nature of these phenomena is discussed as well as possibilities for coherent response. A number of these mechanisms can be equally relevant in the context of linear as well circular accelerators.

INTRODUCTION

The importance of space charge in high intensity beams has primarily two sources: the most common and obvious effect is the incoherent shift of betatron frequencies due to space charge, which may have the effect of pushing particles into the stop-band of a lattice-induced resonance (external nonlinearity); space charge can, however, be a source of resonance itself due to its time dependence (mismatched beams) and/or the nonlinear nature of the space charge force (space charge structure resonances) as well as a self-consistent collective response (space charge driven instabilities). The development of beam halo - important for loss predictions - depends on this complex dynamical behavior, moreover on the six-dimensional initial distribution function as the initial "seed". One usually assumes decoupled longitudinal and transverse phase space distributions, which may not always be the case in real beams in linear accelerators in particular and is difficult to measure. The complexity of these phenomena therefore makes it often difficult to give sufficiently accurate loss predictions. We proceed by discussing some of these mechanism in the order of increasing complexity and refer to linear and circular accelerators whenever appropriate.

INCOHERENT AND COHERENT EFFECTS WITH EXTERNAL NONLINEARITIES

The most obvious effect of space charge is the shift and spread of single particle tunes. For a uniform (KV-distribution) coasting beam space charge leads to a downwards shift of the zero-intensity tune, with a spread around the shift due to amplitude-dependent betatron frequencies for non-uniform distributions (WB, Gaussian etc).

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Note that in linear accelerators external nonlinearities are usually ignored as they normally do not occur with sufficient periodicity - different from internal space charge nonlinearities -, hence this section focusses on circular accelerators. In the presence of such an external nonlinearity, for example an octupole, the space charge shift/spread not only causes a shift and - for sufficiently large spread - a broadening of the resonance condition. In self-consistent simulation one can find coherent phenomena - depending on the distribution function -, which are absent in single particle models. To further discuss the space charge phenomena in this case it is necessary to treat the coasting beam separately from the bunched beam case.

Coasting Beam

For the coasting beam approximation of a circular accelerator we use a set of MICROMAP- simulations in a smooth focusing lattice with an externally applied single octupole as discussed in Ref. [1]. The findings of this study show that the "simple" resonance condition $4Q_x = 25$ does not adequately account for a detailed picture. Results have indicated that the actual response due to the octupole depends significantly on the initial distribution function as well as the coherent response of the beam. Here we compare KV with WB and Gaussian (full tail) distributions noting that the KV case is mostly of academic interest, but it reveals in an accentuated way some of the important mechanisms. The KV response after 1000 turns is shown in Fig. 1 as function of the bare machine tune Q_x. We

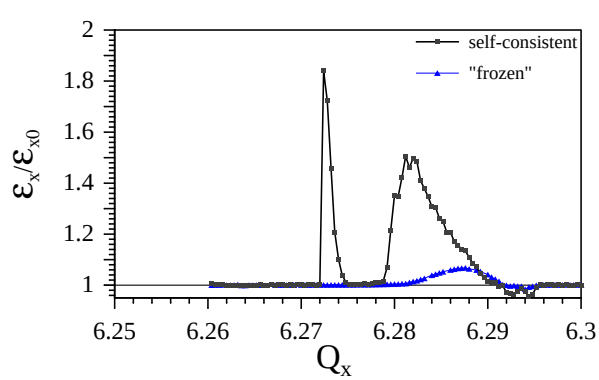

Figure 1: 2D simulation of KV distribution with $\Delta Q_x = -0.045$ and octupole as function of bare machine tune, showing self-consistent and "frozen-in" models (after 1000 turns) (from Ref. [1]).

first discuss the rms emittance growth for a "frozen-in" space charge electric field, where the initial values are not updated, hence the response is entirely incoherent. The absence of response for a distance of the bare machine tune from the resonance line less than 0.045 reflects the incoherent space charge tune shift of the initial KV-beam. The stop-band width of ≈ 0.01 is due to the finite octupole strength. The maximum emittance growth is only $< 6\%$, which is much less than the 25% growth obtained in the same case without space charge. The reason for this is the *detuning* effect of space charge: growing amplitudes weaken the space charge, and the resonance condition gets lost.

The self-consistent simulation, instead, shows a very different behavior with two separate peaks. The broader peak is a direct result of the fourth order resonance, although its height exceeds significantly (more than five times) the maximum "frozen-in" response - a pronounced coherent effect. The perhaps unexpected spike at $Q_x = 6.27$ cannot be explained as direct result of the octupole, but is associated with an envelope instability further discussed in section . Such an envelope instability requires a fractional phase advance of the envelope of half an integer relative to the lattice periodicity as was shown in Refs. [2]. This condition is analogous to the envelope instability in linear accelerators, where a single-particle phase advance above $90°$ per focusing period may induce a half-integer unstable envelope as was first shown in Ref. [3]. Here the "structure period" is absent in the smooth first order lattice, but stems from the local perturbation induced by the relatively strong octupole. The latter occurs at only one position on the circumference, hence the total phase advance of particles per turn is exceeding $6 \times 360° + 90°$.

The rms equivalent waterbag distribution of Ref. [1] shows that the envelope instability peak is unchanged, but the fourth order resonance effect is visibly reduced (Fig. 2). We explain this as a weakening of the coherence induced by the finite tune spread, which is a kind of Landau damping effect. For the Gaussian distribution the picture is still a different one. The much broadened single-particle spectrum of tunes typical for a Gaussian fully overlaps the position of the expected envelope instability frequency, which is therefore effectively "Landau-damped". The much broadened direct response curve is only slightly enhanced compared with the "frozen-in" model (Fig. 3), hence there is an almost complete suppression of any coherent resonance effect. There is also a region of about 0.2% loss, which is practically identical for the self-consistent and "frozen-in" model, and is caused by the Gaussian tails.

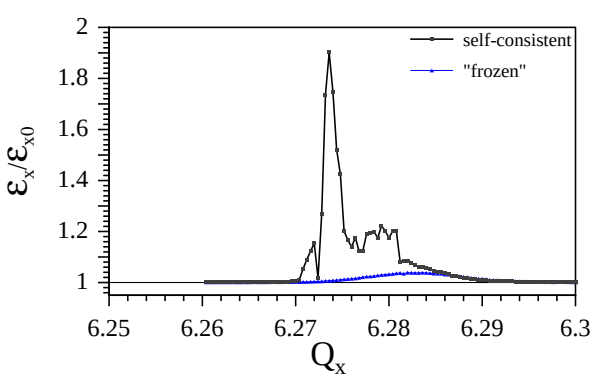

Figure 2: 2D simulation of waterbag distribution (same parameters as Fig. 1) (from Ref. [1]).

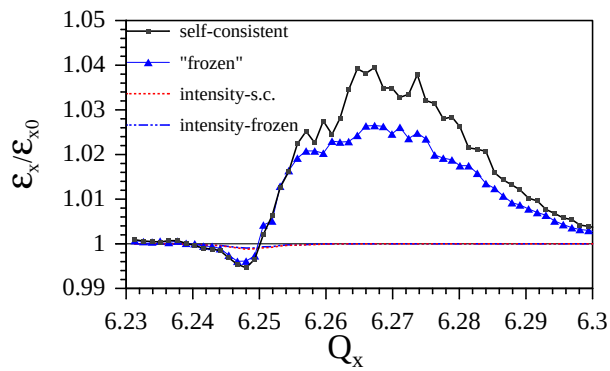

Figure 3: 2D simulation of Gaussian distribution (same parameters as Fig. 1, note the enlarged vertical scale) (from Ref. [1]).

Bunched Beams

The behavior of bunched beams in the presence of an external nonlinearity is quite different from that of coasting beams due to the migration of single particle tunes across the tune footprint. This is caused by the space charge effect varying along the bunch with the synchrotron motion, which must be added to a similar effect due to finite chromaticity and momentum spread. For sufficiently large synchrotron frequency this may have a phase mixing effect - similar to that of the above discussed coasting beam Gaussian distribution - and suppress the possibility for coherent space charge response in a coasting beam. One may therefore expect a primarily incoherent response for bunched beams.

A detailed analysis of the combined effect of space charge, an external octupole and synchrotron motion has been studied in an experiment at the CERN Proton Synchrotron [4]. Long term emittance growth and beam loss have been compared with computer simulation and explained successfully in terms of the cumulative effect of many crossings of particles across the resonance. A similar experiment was repeated recently with an external sextupole at the SIS18 of GSI with similar findings [5]. The simulation model for these long-term simulations ($> 10^5$ turns) has been using a "frozen-in" space charge under the assumption that coherent response is absent.

MISMATCH DRIVEN RESONANCES AND HALO

A purely space charge driven resonant mechanism common to linear and circular accelerators is the well-known halo formation as a result of an envelope mismatch. Such a mismatch is usually the result of an injection error or a sudden structure change. It has been originally discussed in a simplified core-test particle (CTP) model, where a 2:1 (parametric) resonance between the mismatch envelope oscillation and individual particles was described as a major source of amplitude growth and halo formation [6]. The resonant force was assumed to be the space charge force of the oscillating beam core.

Due to space charge tune depression single particles inside the core oscillate slower than half the (also space charge shifted) envelope frequency, therefore only particles initially outside of the core (in the tails of the distribution function) have a chance to satisfy the required 2:1 frequency relationship between core and particle. Therefore quantitative halo predictions always suffer from a poor knowledge of the initial tails, moreover, the oscillating core may reveal a larger spectrum of frequencies to be taken into account (see Ref. [7] and a more recent and detailed study in Ref. [8]). Further practically important issues are the self-consistent evolution of an rms mismatched beam, which may lead to a coherently driven break-up of the initial core distribution as first discussed in Ref. [9]; as well as important anisotropic effects due to different frequencies and/or emittances between the transverse and transverse-longitudinal degrees of freedom, which may give rise to unbounded halo [10].

All these mismatch induced processes have in common that the electrostatic field energy contained in the mismatch oscillation is "thermalized" by the halo evolution [11]. The term "thermalization" is actually misleading, since the halos discussed here usually extend significantly beyond Gaussian tails. It can be shown that even for realistic lattices of linear accelerators the - then anisotropic field energy - gives an upper bound for the rms emittance growth by the induced halo [10],[12]. The actual radial extent of the halo as well as the time-scale over which it forms depends, among others, on whether the starting distribution already has a tail or not; it may take very long for a well-truncated initial distribution. Also it is largely unexplored what the effect of realistic correlations between different phase planes in the initial distribution are (keeping in mind that they are hard if not impossible to measure).

SPACE CHARGE STRUCTURE RESONANCES AND INSTABILITIES

In this section we assume a sufficiently well rms-matched beam in a perfectly linear and periodic lattice, either circular or linear. In the absence of intrabeam scattering or noise the remaining source of emittance or amplitude growth is space charge itself.

A significant effect on the emittance requires a resonant action, which only occurs if a proper resonance condition is fulfilled. As space charge self-interaction is not a single-particle effect, such a resonance condition cannot simply be described just by a single particle structure resonance condition of the kind $nQ = N$, where N is the number of super-periods or equal cells per turn (in a linac correspondingly $\sigma = 360/n$. As was shown in Ref. [2] this problem has the following complexity:

- a coherent frequency shift is needed to express the space charge coherent interaction and corresponding shift of the stop-band
- the driving force may be present in the initial beam for a sufficiently nonuniform initial distribution (structure *resonance*)
- alternatively the driving force may grow exponentially from an initial noise level (structure *instability*)

To demonstrate these mechanisms we refer to the example of a ring lattice with 12 identical triplet focussing cells in Ref. [2], where initially an rms matched waterbag distribution was used and an emittance ratio ϵ_x/ϵ_y=4. In the self-consistent simulation the bare machine tunes have been kept fixed at $Q_{0x} = 4.9$ and $Q_{0y} = 3.4$, corresponding to a phase advance per cell of $\sigma_{0x} = 147^0$ and $\sigma_{0y} = 102^0$. With increasing intensity an rms emittance growth in y was found in the region, where the rms value of the space charge depressed tune in y was $2.7 < Q_y < 2.95$ ($81^0 < \sigma_y < 88.5^0$) as shown in Fig. 4. Two distinct mech-

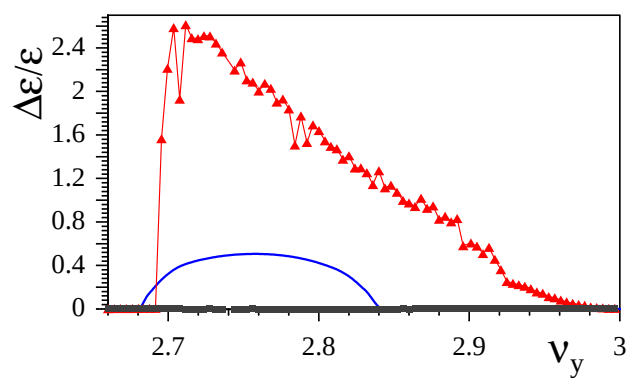

Figure 4: Saturated r.m.s. emittance effect after 200 turns in an ideal linear lattice with (Q_{0x},Q_{0y})=(4.9,3.4) as function of Q_y. Also shown is 10x(λ-1) of the theoretical envelope instability growth factor λ (fromRef. [2]).

anisms are expected to play a role in this stop-band. The first one is a structure envelope instability, which can be described by the following resonance equation using rms values of the space charge depressed tunes

$$2Q_y + \Delta\omega_2 = 12/2, \tag{1}$$

or in linac notation:

$$\sigma_y + \Delta\omega_2' = 90^0. \tag{2}$$

It is characterized by a phase advance of 180^0 of the underlying envelope mode per focusing period, hence a 1:2 (half-integer) relationship between envelope mode and lattice periodicity. The envelope instability was first studied in Ref. [3] and extensively reviewed - with a number of possible ramifications - in Ref. [13]. Note that we have added a (positive) "coherent resonance shift" in Eqs. 1,2 to account for the fact that the center of the stop-band of this instability does not simply occur at the resonance conditions $2Q_y = 12/2$ or $\sigma_y = 90^0$ (with rms tune values), but it is shifted downwards. The theoretically expected stop-band for the particular lattice is obtained by solving the KV envelope equations for the growth factors of the unstable envelope modes. As shown in Fig. 4, the particular linear lattice with zero-current phase advance per cell of $\sigma_{0y} = 102^0$ shows an envelope instability in the region $2.68 < Q_y < 2.84$, or $80.4^0 < \sigma_y < 85.2^0$with an associated envelope growth per cell by a factor λ.

The remaining part of the stop-band is due to the structural fourth order resonance, which can be written as

$$4Q_y + \Delta\omega_4 = 12, \tag{3}$$

or

$$\sigma_y + \Delta\omega_4' = 90^0 \tag{4}$$

in linac notation. Here the driving force is the fourth order term in the space charge potential due to the initial WB, modulated by the periodic focusing structure. Again, a coherent tune shift is employed to demonstrate the downwards shift of the center of the stop-band. The fourth order nature in this part of the stop-band is shown in the phase space plot of Fig. 5 for $Q_y = 2.86$ (in Fig. 4) at an early and a more advanced stage.

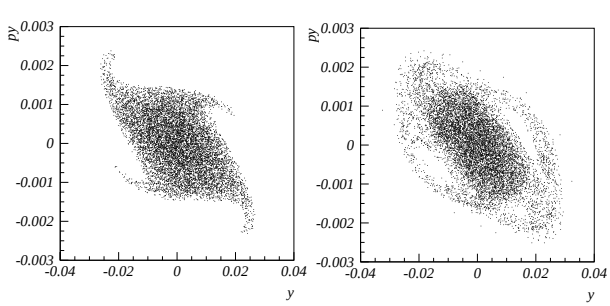

Figure 5: Phase space projections in y for fourth order structural response and Q_y=2.86 after turn 2 and 30 (from Ref. [2]).

SCALING LAWS FOR SPACE CHARGE STRUCTURE RESONANCES

It was recently shown that for at least two candidates of space charge structure resonances, the emittance exchange "Montague resonance" and the fourth order structure resonance, a common feature exists in terms of very similar scaling laws predicting the rms emittance growth for complete crossing of the stop-band.

Montague Resonance

The Montague resonance is a difference resonance driven by the zeroth harmonic of the fourth order term of the space charge potential. The condition for the stop-band center can be written with a coherent tune shift $\Delta\omega_{2,2}$, which is negative for $\epsilon_x > \epsilon_y$, and positive for $\epsilon_x < \epsilon_y$ [14]:

$$2Q_x - 2Q_y + \Delta\omega_{2,2} = 0. \tag{5}$$

In linacs the emittance difference is usually between longitudinal and transverse, hence the appropriate resonance condition is written as (again a negative coherent shift for $\epsilon_z > \epsilon_x$):

$$\sigma_z - \sigma_x + \Delta\omega_{2,2}' = 90^0. \tag{6}$$

Following Ref. [15] the starting point for a scaling law is an analytical expressions for the stop-band width, which was derived in Ref. [14]. Writing the stop-band in terms of a spread of the horizontal tune, the result is:

$$\Theta = \frac{3}{2}(\sqrt{\epsilon}_r - 1)\Delta Q_x, \tag{7}$$

where ΔQ_x is the incoherent space charge tune spread defined as maximum tune spread of a Gaussian beam, i.e. twice the KV-equivalent tune shift; ϵ_r is the initial ratio of ϵ_x and ϵ_y (here assumed ≥ 1 without loss of generality).

For fixed ΔQ_x the emittance growth after crossing is found inversely proportional to the number of turns/cells per crossing, except for large exchange, where the emittances go into saturation towards full exchange. This is shown in Ref. [14] for a crossing over the range $5.15 \leq Q_{0,x} \leq 5.27$ enclosing the stop-band, while $Q_{0,y}$ is kept fixed at the value 5.21. For this crossing "from below" the final emittances after crossing the band at variable number of turns is shown in Fig. 6.

This finding, together with the observation that the number of turns needed for a certain emittance exchange is inversely proportional to ΔQ_x, whereas the stop-band width is directly proportional to ΔQ_x, explains that the result of a crossing can be written in terms of the "scaling parameter" S, with $\dot{Q}$ the change of tune per turn:

$$S \equiv \frac{(\Delta Q_x)^2}{\dot{Q}}. \tag{8}$$

According to the simulation results of Ref. [15] this leads to a scaling law of the form

$$\frac{\Delta\epsilon_y}{\epsilon_y} = \alpha_{2,2}(\sqrt{\epsilon}_r - 1)^2 S, \tag{9}$$

where $\alpha_{2,2} \approx 0.5$.

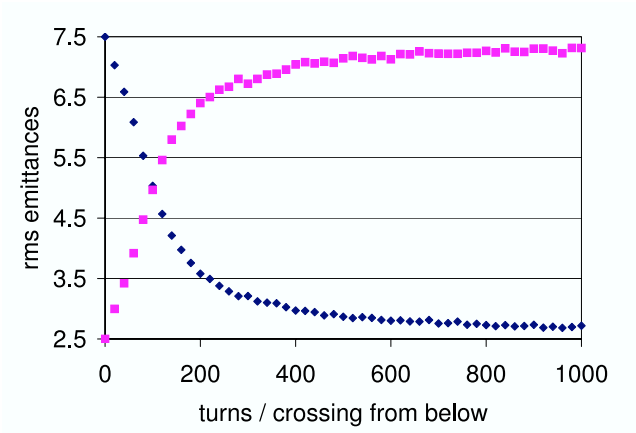

Figure 6: Final emittances after crossing the stop-band at variable number of turns ($\Delta Q_y = -0.105$) (from Ref. [14]).

Fourth Order Structure Resonance

Following Ref. [15] the fourth order structure resonance for downwards crossing follows a similar scaling, but not with the same power in S:

$$\frac{\Delta\epsilon_y}{\epsilon_y} = \alpha_{0,4} S^n. \tag{10}$$

In most simulations the power was found to be $n = 2$, except for small emittance growth, where a lower power seemed more appropriate. $\alpha_{0,4}$ has to be determined for a given lattice by fitting to numerical results.

Fourth order and sixth order structure resonances in rings have been applied to non-scaling FFAG accelerators, where a number of them are crossed due to the large tune swing [16]. It has also been shown that they may even by excited by lattice errors [17].

The relevance to linacs has been demonstrated by a recent simulation study of 3D bunches in a drift tube linac, where the fourth order structure resonance is also found with a similar scaling law [18].

CONCLUDING REMARKS

We have summarized work showing that transverse space charge may lead to a diversity of resonant effects beyond the "trivial" incoherent tune shift, which could be equally important for linear as well as circular accelerators: mismatch resonant behavior; the possibility of structure resonances only driven by space charge; the appearance of envelope instabilities; coherent shifts describing the shift of resonance stop-bands; and the existence of common scaling laws for resonance crossing. In practical applications involving significant space charge all these effects may appear in combination with each other. Furthermore, structurally driven space charge effects may be a result of the regular lattice structure, but also - though weaker - of errors in the lattice. This overview cannot claim completeness, and many questions are left open. The role of the 6D distribution function, including in particular possible correlation effects between the planes, is one of these topics that might deserve further study.

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REFERENCES

[1] I. Hofmann, G. Franchetti, J. Qiang and R.D. Ryne, 29th ICFA Workshop *Halo'03*, Montauk, 2003.

[2] I. Hofmann, G. Franchetti and A. Fedotov, AIP Conference Proceedings AIP-642, p. 248 (2002).

[3] I. Hofmann, L.J. Laslett, L. Smith, I. Haber, Part. Acc. **13**, 145 (1983).

[4] G. Franchetti, *et al.*, Phys. Rev. ST Accel. Beams **6**, 124201 (2003).

[5] G. Franchetti, *et al.*, paper WGA22, these proceedings.

[6] C.K. Allen and T.P. Wangler, PRSTAB 5, 124202 (2002).

[7] J.M. Lagniel, Nucl. Instr. Meth. A 345, 405 (1994).

[8] S.M. Lund et al., Nucl. Instr. Meth. A577, 173 (2007).

[9] R.L.Gluckstern, Phys. Rev. Letters 73, 1247 (1994).

[10] G. Franchetti et. al., Phys. Rev. Lett. 88, 254802 (2002).

[11] M. Reiser, *Theory and Design of Charged Particle Beams*, Wiley, New York, 1994.

[12] I. Hofmann, G. Franchetti, J. Qiang, R. Ryne,F. Gerigk, D. Jeon and N. Pichoff in *Proceedings of the European Accelerator Conference*, Paris, 2002, ed. J.L. Laclare, p. 74 (2002).

[13] S.M. Lund et al., PRSTAB, Vol. 7, 024801 (2004).

[14] I. Hofmann and G. Franchetti, Phys. Rev. ST Accel. Beams **9**, 054202 (2006).

[15] I. Hofmann and G. Franchetti, PAC07, paper THPAN017 (2007).

[16] S.Y. Lee, G. Franchetti, I. Hofmann, F. Wang and L. Yang, New J. Phys. **8**, No 11, 291 (2006).

[17] X. Pang et al., paper WGA21, these proceedings.

[18] D. Jeon, paper WGB12, these proceedings.